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Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - CARRY OVER THEORY EXAMINATION (2023 - 2024)

Subject: Mathematical Foundations – II

Time: 3 Hours Max. Marks: 100

General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A 20

1. Attempt all parts:-

1-a. Evaluate the integral $\int_0^3 \int_1^2 xy(1+x+y)\,dy\,dx$ (CO1) 1

(a) 122/3

(b) 120/4

(c) 123/4

(d) None of these

1-b. Evaluate $\left|-\frac{3}{2}\right|$ (CO1) 1

(a) $\frac{\sqrt{\pi}}{3}$

(b) $\frac{4\sqrt{\pi}}{3}$

(c) $-\frac{3\sqrt{\pi}}{4}$

(d) None of these

1-c. General solution of the differential equation $(D^3-2D+4)y=0$ is (CO 2) 1

(a) $c_1e^{-2x}+x(c_2\cos x + c_3\sin x)$

(b) $c_1e^{-2x}+e^{x}(c_2\cos x + c_3\sin x)$

(c) $c_1e^{-2x}+e^{-x}(c_2\cos x + c_3\sin x)$

(d) $c_1e^{-2x}+e^{2x}(c_2\cos x + c_3\sin x)$

1-d. 1

On putting $x = e^z$, the transformed differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x$ is (CO 2)

(a) $\frac{d^2y}{dz^2}-y=e^z$

(b) $\frac{d^2y}{dz^2}+y=e^z$

(c) $\frac{d^2y}{dz^2}-2y=e^z$

(d) $\frac{dy}{dz}-2y=e^z$

1-e. The degree of the equation $z^2 = pqxy$ is (CO3) 1

(a) 1

(b) 2

(c) 3

(d) None of these

1-f. Which of the following is the correct partial differential equation of the relation z = (x + a)(y + b), where 'a' and 'b' are constant (CO3) 1

(a) $z=pq$

(b) $z=p+q+ab$

(c) $z=px+qy+pq$

(d) $z=px+qy-pq$

1-g. 1

Inverse Laplace of the function $f(s)=\left[\frac{1}{s(s^2+1)}\right]$ is (CO 4)

(a) $1-\cos t$

(b) $1+\sin t$

(c) $1-\sin t$

(d) None of these

1-h. 1

Inverse Laplace of the function $f(s)=\frac{1}{2s+3}$ is (CO4)

(a) $-\frac{1}{2}e^{-3t/2}$

(b) $-\frac{1}{2}e^{3t/2}$

(c) $\frac{1}{2}e^{-3t/2}$

(d) None of these

1-i. The ratio of present ages of Sri and Gowtham is 3: 4. Mahesh is 6 years older than Sri and two years younger than Gowtham. The sum of the present ages of Sri and Mahesh is (CO5) 1

(a) 48 years
(b) 50 years
(c) 52 years
(d) 54 years

1-j. In what time will be Rs 2000 amount to Rs 2500 at 10% per annum simple interest? (CO5) 1

(a) 2.5 years
(b) 1.5 years
(c) 2 years
(d) 5 years

2. Attempt all parts:-

2.a. Evaluate Integral $\int_1^2 \int_0^x \frac{dy\,dx}{x^2+y^2}$. (CO1) 2

2.b. Find the particular integral of differential equation $(D^2+4D+8)y = \sin(2x+3)$. (CO2) 2

2.c. Solve the partial differential equation $\frac{\partial^2 z}{\partial x\,\partial y} = 0$. (CO3) 2

2.d. Find the inverse Laplace transform of the function $f(s) = \frac{1}{s^2+3}$. CO 4 2

2.e. Pointing to a photograph, Rahul said, "She is the mother of the wife of my brother's father". How is the lady in the photograph related to Rahul? (CO5) 2

SECTION-B 30

3. Answer any five of the following:-

3-a. Evaluate $\int_0^1 \int_{y^2}^{y} (1+xy^2)\,dx\,dy$. (CO1) 6

3-b. Prove that $\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$ where m > 0, n > 0. (CO1) 6

3-c. Solve the differential equation $(D^2+D+1)y = \sin 2x$. (CO 2) 6

3-d. Solve $\frac{dx}{dt}+y = \sin t$, $\frac{dy}{dt}+x = \cos t$, given that $x = 2$ and $y = 0$ when $t = 0$. (CO2) 6

3.e. Solve the partial differential equation $x(y-z)p+y(z-x)q = (x-y)$. (CO3) 6

3.f. Evaluate the value of the integral $\int_0^{\infty} e^{-2t} \sin^3 t\ dt$. CO 4 6

3.g. (i) Lalit along with his family decided to take a road trip to a nearby resort and spend the weekend there. He started from his home and from there drove 70 km to 6

the south, he then took a right turn and drove 30 km. Next, he took a right turn and drove 30 km and stopped at a restaurant. What is the shortest distance between his house and the restaurant?

(ii) The compound interest on a certain sum of money at a certain rate for 2 years is Rs. 40.80 and the simple interest on the same sum is 40 at the same rate and for the same time. Find the rate of interest per annum? (CO5)

SECTION-C 50

4. Answer any one of the following:-

4-a. 10

Change the order of integration for $I = \int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ and evaluate the same. (CO1)

4-b. 10

The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B and C. Apply Dirichlet's integral to find the volume of the tetrahedron OABC. Also find its mass if the density at any point is kxyz. (CO1)

5. Answer any one of the following:-

5-a. Solve the differential equation by reducing it into normal form (removal of first derivative) $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}\sin 2x$. CO 2 10

5-b. Solve the differential equations by method of variation of parameters $y'' - y = \frac{2}{1+e^x}$. CO 2 10

6. Answer any one of the following:-

6-a. Solve the following partial differential equation 10

$(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x - y)$. (CO3)

6-b. Solve $\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = e^{2x+3y} + \sin(x - 2y)$. (CO3) 10

7. Answer any one of the following:-

7-a. 10

If $L^{-1}\{f(s)\} = F(t)$, show that $L^{-1}\{\frac{1}{s}f(s)\} = \int_0^t F(x)dx$. Also find $L^{-1}\left\{\frac{1}{s^2(s+a)}\right\}$. CO 4

7-b. Solve the following simultaneous differential equations by using Laplace transform $\frac{dx}{dt} + 4\frac{dy}{dt} - y = 0, \frac{dx}{dt} + 2y = e^{-t}$, *Given that* $x(0) = y(0) = 0$. CO 4 10

8. Answer any one of the following:-

8-a. (i)Study the information given below carefully and accordingly answer the following questions: Akshat is going shopping at a mall nearby with his friends. They start from point X and move 15 km to the north. They then take a right turn and move 20 km, followed by a left turn. After moving for another 40 km, they turn left and walk 20 km. 10

(a) In which direction is Akshat along with his friends with reference to point X?

(b) What is the distance between point X and the final destination?

(ii) In a room with a four students, Q is sitting to the North of P. R is in the East of Q and S is to the left of P. Based on the given information, in which direction is S sitting with respect to R? (CO5)

8-b. (i) The present ages of three persons, Raj, Rajesh and Ravi is in proportions 4: 7: 9. Eight years ago, the sum of their ages was 56. What is the present age of Ravi? 10

(ii) The present age of Aradhana and Aadrika is in the ratio 3:4. 5 years back, the ratio of their ages was 2:3. What is the present age of Aradhana? (CO5)